5^{th} Lecture of Operation Research 2

Changes affecting feasibility:

- Change R.H.S of a constraint.
- Addition of a new constraint.

Feasible range of b_i Where b_i : is the R.H.S of the constraint.

$$b_i \rightarrow b_i + d_i$$

$$\begin{bmatrix} New \\ Sol. \\ column \end{bmatrix} = \begin{bmatrix} & IM & \end{bmatrix} \begin{bmatrix} b_1 \\ b_i + d_i \\ b_m \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \geq 0$$

Find the feasible range of b_2 where:

$$\begin{bmatrix} Orginal \\ Sol. \\ ution \\ column \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 1 \\ 2 \end{bmatrix} \quad , \quad IM = \begin{bmatrix} 2/3 & -1/3 & 0 & 0 \\ -1/3 & 2/3 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -2/3 & 1/3 & 0 & 1 \end{bmatrix}$$

To get the range we will exchange b_2 with $b_2 + d_2$

$$\begin{bmatrix} New \\ Sol. \\ col \\ umn \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 & 0 & 0 \\ -1/3 & 2/3 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -2/3 & 1/3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 8 + d_2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} (4 - \frac{8}{3} - \frac{d_2}{3}) \\ (-2 + \frac{16}{3} + \frac{2d_2}{3}) \\ (-6 + 8 + d_2 + 1) \\ (-4 + \frac{8}{3} + \frac{d_2}{3} + 2) \end{bmatrix}$$

The solution is feasible when $\begin{bmatrix} (4 - \frac{8}{3} - \frac{d_2}{3}) \\ (-2 + \frac{16}{3} + \frac{2d_2}{3}) \\ (-6 + 8 + d_2 + 1) \\ (-4 + \frac{8}{3} + \frac{d_2}{3} + 2) \end{bmatrix} \geq 0$

يبقى هأخد كل عنصر من عمود الـ Solution الجديد اللي طلعلي وهخليه أكبر من او يساوى الصفر واطلع الـ Range الـ d_2 ومنه هطلع الـ Range الـ b_2 .

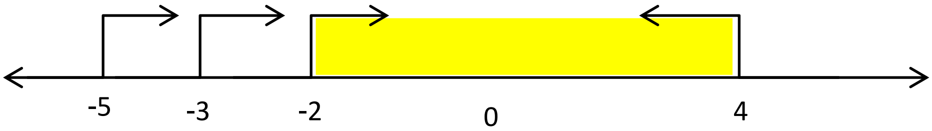
$4 - \frac{8}{3} - \frac{d_2}{3} \geq 0 \quad \rightarrow \quad d_2 \geq -3 \leq 4$

$-2 + \frac{16}{3} + \frac{2d_2}{3} \geq 0 \quad \rightarrow \quad d_2 \geq -5$

$-6 + 8 + d_2 + 1 \geq 0 \quad \rightarrow \quad d_2 \geq -3$

$-4 + \frac{8}{3} + \frac{d_2}{3} + 2 \geq 0 \quad \rightarrow \quad d_2 \geq -2$

هرسم المسألة على خط الاعداد واشوف الـ d_2 محصورة بين كام وكام



Then the range of d_2 will be :

$$-2 \leq d_2 \leq 4$$

Then the range of b_2 will be :

$$-2+8 \leq b_2 \leq 4+8$$

$$6 \leq b_2 \leq 12$$

2- Addition of a new constraint (not redundant constraint) :

اولا هشوف الـ Optimal Solution اللى عندى لو بيحقق الـ Constraint الجديد اللى أنا ضفته يبقى الـ Constraint اللى أنا ضفته لم يؤثر على الـ Feasibility والحل اللى عندى Still Feasible .

Example no. 2:

Basic	X1	X2	S1	S2	S3	S4	Sol.
Z	0	0	1/3	4/3	0	0	38/3
X2	0	1	2/3	-1/3	0	0	4/3
X1	1	0	-1/3	2/3	0	0	10/3
S3	0	0	-1	1	1	0	3
S4	0	0	-2/3	1/3	0	1	2/3

Find the effect of adding a new constraint:

a) $X_1 \leq 4$

هشوف الـ Optimal solution اللى عندى بيحقق الـ Constraint ولا لاء

عندى الـ $X_1 = 10/3$

10/3 ≤ 4

Then the solution still feasible.

$$X_1^* = 10/3 \qquad X_2^* = 4/3 \qquad Z^* = 38/3$$

b) $X_1 \leq 3$

$$10/3 \nleq 3$$

Then the current O.S doesn't satisfy the additional constraint and the new solution is not feasible.

$X_1 + S_5 = 3$

هبدأ أضيف الـ Constraint الجديد فى الجدول وابدأ أحل Dual Simplex علشان أحل مشكلة الـ feasibility

Basic	X1	X2	S1	S2	S3	S4	S5	Sol.
Z	0	0	1/3	4/3	0	0	0	38/3
X2	0	1	2/3	-1/3	0	0	0	4/3
X1	1	0	-1/3	2/3	0	0	0	10/3
S3	0	0	-1	1	1	0	0	3
S4	0	0	-2/3	1/3	0	1	0	2/3
S5	0	0	1/3	-2/3	0	0	1	-1/3
Z	0	0	1	0	0	0	2	12
X2	0	1	½	0	0	0	-1/2	3/2
X1	1	0	0	0	0	0	1	3
S3	0	0	-1/2	0	1	0	3/2	5/2
S4	0	0	-1/2	0	0	1	½	½
S2	0	0	-1/2	1	0	0	-3/2	½

The New Optimal solution:

$X_1^* = 3$ $X_2^* = 3/2$ $Z^* = 12$

Changes affecting optimality :

1. Change in objective function coefficient (C_j).

Home Work

Find the new optimal solution:

Basic	X1	X2	S1	S2	S3	S4	Sol.
Z	0	0	1/3	4/3	0	0	38/3
X2	0	1	2/3	-1/3	0	0	4/3
X1	1	0	-1/3	2/3	0	0	10/3
S3	0	0	-1	1	1	0	3
S4	0	0	-2/3	1/3	0	1	2/3

a. if Objective Fun. Max Z = 3 X_1+ 2 X_2 becomes

Max Z = 5 X_1+ 4 X_2 + 0 S_1 + 0 S_2 + 0 S_3 + 0 S_4

$$(Y_1\ \ Y_2\ \ Y_3\ \ Y_4) = (4\ \ 5\ \ 0\ \ 0)\begin{bmatrix} 2/3 & -1/3 & 0 & 0 \\ -1/3 & 2/3 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -2/3 & 1/3 & 0 & 1 \end{bmatrix} = (1\ \ 2\ \ 0\ \ 0)$$

$Y_1^* = 1$ $Y_2^* = 2$ $Y_3^* = 0$ $Y_4^* = 0$

b. if Objective Fun. Max Z = 3 X_1+ 2 X_2 becomes

Max Z = 4 X_1+ 1 X_2 + 0 S_1 + 0 S_2 + 0 S_3 + 0 S_4

$$(Y_1 \;\; Y_2 \;\; Y_3 \;\; Y_4) = (1 \;\; 4 \;\; 0 \;\; 0) \begin{bmatrix} 2/3 & -1/3 & 0 & 0 \\ -1/3 & 2/3 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -2/3 & 1/3 & 0 & 1 \end{bmatrix} = (7/3 \;\; 0 \;\; 0 \;\; -2/3)$$

Y_1^*= 7/3 $\qquad$ Y_2^* = 0 $\qquad$ Y_3^* = 0 $\qquad$ Y_4^* = -2/3

c. Find the range of C_1 to keep the solution optimal .

Max Z = 3 X_1+ 2 X_2 + 0 S_1 + 0 S_2 + 0 S_3 + 0 S_4

$$(Y_1 \;\; Y_2 \;\; Y_3 \;\; Y_4) = (2 \;\; 3 + d_1 \;\; 0 \;\; 0) \begin{bmatrix} 2/3 & -1/3 & 0 & 0 \\ -1/3 & 2/3 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -2/3 & 1/3 & 0 & 1 \end{bmatrix} = \left(\frac{1}{3} - \frac{d_1}{3} \, , \, \frac{4}{3} + \frac{2d_1}{3} , \;\; 0 \, , \;\; 0 \right)$$

أنا عايز الـ Range اللى الحل يفضل فيه Optimal والمسألة عندى Maximization يبقى هجيب معاملات الـ Variable اللى مش موجودين فى عمود الـ Basic ولازم يكونوا يا صفر يا موجب علشان المسألة Max يبقى المعاملات اللى هجيبها هخليها أكبر من او يساوى الصفر واجيب الـ Range الـ d_1 ومنه هجيب Range الـ C_1

Coeff. Of S_1 in z-row:

$$Y_1 - 0 = \frac{1}{3} - \frac{d_1}{3}$$

$$\frac{1}{3} - \frac{d_1}{3} \geq 0 \qquad d_1 \; \leq 1$$

Coeff. Of S_1 in z-row:

$$Y_2 - 0 = \frac{4}{3} + \frac{2d_1}{3}$$

$$\frac{4}{3} + \frac{2d_1}{3} \geq 0 \qquad d_1 \; \geq -2$$

$$-2 \; \leq \; d_1 \; \leq 1$$

$$3 - 2 \leq C_1 \; \leq 3 + 1$$

$$1 \leq C_1 \; \leq 4$$

Best Wishes